The Combined Task Allocation and Path Finding Problem

Christian Henkel[1] and Marc Toussaint[2]

***Abstract*— In logistics scenarios it is common to use multiple robots that have to transport multiple goods. These goods will commonly require to be picked up at one location and dropped of at a different location.**

We introduce and formalize the Combined Task Allocation and Path Finding problem, which combines the problems of transport task allocation and multi agent path finding. We show, that only solving both combined problems allows optimality for the overall system while avoiding collisions between agents at runtime. This necessity of combined solving is demonstrated by an example.

I. Introduction

In Multi-Agent Path Finding (MAPF) the commonly studied problem is defined as a set of agents with one start and goal state each. Potential applications of MAPF in real-world scenarios often extend the problem to a bigger configuration space. Especially in transport scenarios, tasks are not necessarily linked to a predefined agent. In this paper, we are considering transport tasks that have a start and goal state with no predefined allocation of agents. We call this Combined Task Allocation and Path Finding (CTAPF). It extends the configuration space by the choice of agent for each task. This offers additional potential in terms of collision avoidance, because agents can not only re-plan, but also switch to other tasks in order to solve the collision-free MAPF problem. It also saves computation time, because by combining the two problems, synergies for single-agent path computations occur.

Our method is applicable in any multi-agent domain concerned with collisions between agents and with free task allocation, where tasks can be described as the traversal from one state to another. Such domains may be found in the field of

a) Autonomous Guided Vehicles (AGVs) for intralogistics,
b) general logistics,
c) multi-manipulator industrial robots,
d) fetch and carry tasks for household multi-robot systems.

II. Related Work

The task allocation problem is a well researched challenge in the field of multi-agent systems and operations research. We refer to it as Multi-Agent Task Allocation (MATA). A centralized approach to solve it is the Hungarian algorithm [1].

[1]Fraunhofer Institute for Manufacturing Engineering and Automation IPA, Stuttgart, Germany `cch@ipa.fhg.de`

[2]Machine Learning and Robotics Lab, University of Stuttgart, Germany `marc.toussaint@informatik.uni-stuttgart.de`

The particular problem of transport task allocation is called Pickup and Delivery Problem (PDP) [2] in operations research. See [3] for a survey article.

Multi-Agent Path Finding (MAPF) is another intensively studied problem in multi-agent systems [4]. The decision in MAPF concerns how a number of agents will be traveling to their goal poses without colliding, also an NP-hard problem [5]. Solving the problem with collision avoidance at runtime can lead to deadlocks especially in narrow environments as discussed by [6] and more recently by [7].

One problem formulation that is more closely related to transport systems is the Combined Target-Assignment and Path-Finding (TAPF) introduced by Ma and Koenig [8]. It first solves the assignment problem of target locations to agents before solving the MAPF problem. Instead of single goals we consider the whole transport task allocation.

The joint solution of MATA and MAPF, that we are proposing, was previously studied in [9], where the problem is considered as Mixed Integer Linear Programming (MILP) but due to collisions on the single-agent path level we think it should be considered a Mixed Integer *Non-Linear* Programming (MINLP) Problem. Therefore, the solver proposed by Koes et al. can not solve the problem optimally since it ignores agent-agent collisions.

In the literature we can find many approaches that go towards solving the problems of MATA and MAPF in interesting settings. Little research is done in combining both which is the core aspect addressed in this paper. To our knowledge it is the first to solve the combined MATA and MAPF problem optimally.

III. Problem Formulation

As discussed above, MAPF and MATA have been researched intensively. We now propose a combined problem formulation.

We consider a configuration space $\mathscr{X}$, and n agents with joint configuration state $x = (x_1, .., x_n) \in \mathscr{X}^n$.

We have a set $t = \{t_1, t_2, ..., t_m\}$ of m tasks, each one consists of a start x_i^s and a goal x_i^g configuration, $t_i = (x_i^s, x_i^g) \in \mathscr{X}^2$ with $i \in (1, ..., m)$.

Once an agent has reached a task's start pose, it is automatically assigned to the task, and we say the task is *running*. It is fixed to this task and can not be reassigned to another task before reaching its goal.

The problem is to find a minimal duration $T \in \mathbf{N}$ and paths P_j of length T for each agent j such that every task it fulfilled. In the following we more formally define the constraints on the path (that they follow a roadmap and do not collide), and when tasks are fulfilled.

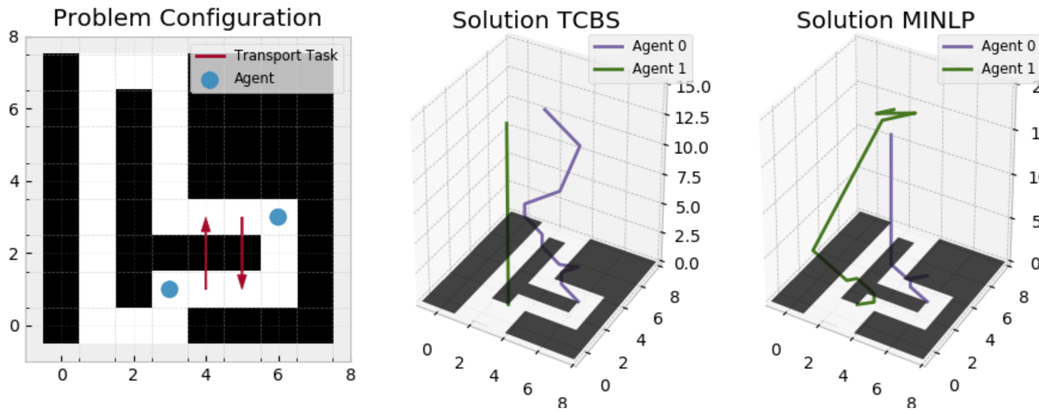

Fig. 1. Example problem configuration (left) and solution as path set in space-time by optimal TCBS planner (middle) and by separately solving task allocation as MINLP optimization problem and path finding with CBS (right).

Each path $P_j = (P_j^0, .., P_j^T)$ is a sequence of $T+1$ configurations, where $P_j^0 \in \mathscr{X}$ is constrained to be the start configuration of agent j, and $P_j^{t+1} \in \partial(P_j^t) \subseteq \mathscr{X}$ is adjacent to the agents previous configuration P_j^t. Adjacency is defined by a roadmap $\partial(\bullet)$ which, for each $x \in \mathscr{X}$ defines its neighborhood $\partial(x) \subseteq \mathscr{X}$.

Given the set of paths $\{P_1, .., P_n\}$ we require them to be non-colliding, meaning that $\nexists t, j, k : P_j^t = P_k^t$. In addition, we also require agent movements to be non-colliding, that is, they must not swap places on the roadmap, $\nexists t, j, k : P_j^{t+1} = P_k^t \wedge P_j^t = P_k^{t+1}$.

Given a path P_j of an agent j, we can compute the implicit assignment $\tau_j^T \in \{1, .., m\}$ of agent j to a task $i \in \{1, .., m\}$ at time step T. The mapping from P_j to τ_j is unique given the rule that a task becomes automatically active if the agent visits its start configuration and ends only when it reaches the task goal. Based on this it is also uniquely defined whether a task i is fulfilled by a path P_j. Therefore, given all agent paths P_j we can evaluate whether all tasks are fulfilled.

In summary, input to the problem are the agent's initial configurations $\{P_1^0, .., P_n^0\}$, the roadmap defined by $\partial(\bullet)$, and the set of tasks t. Output is the total time T and all agent paths $\{P_1, .., P_n\}$, subject to the path constraints and fulfillment of all tasks. The objective is to minimize T.

IV. Evaluation

A. Example Problem and Solution

Figure 1 shows an example of one problem configuration and the resulting path set after solving the problem. It is possible to compare the two solutions by and optimal planner, TCBS (middle) and MINLP [9] (right). From the paths in the pictures follow the task execution times displayed in Table I per task.

It is visible that the optimal planner TCBS produces a solution quality superior to MINLP while it does only utilize one agent. The task allocation step of MINLP assigns each agent to the closest task. When the tasks are executed, the two agents in MINLP block each other. In this sense the TCBS algorithm successfully finds a solution that takes the transport task execution into account. This shows how special cases require to jointly solve the MATA and the MAPF problems which is the main claim of this paper.

Algorithm	Task 1	Task 2	Sum
TCBS (optimal)	6	15	21
MINLP	6	20	26

TABLE I

EXECUTION TIMES FOR THE TASKS IN THE EXAMPLE.

V. Conclusion

Combined Task Allocation and Path Finding (CTAPF) is a practically highly relevant problem. We provided a detailed formalization of the problem. In comparison to other approaches is demonstrated how the combined solving of task allocation and path finding improves the solution quality over solutions that solve them separately.

References

[1] H. Kuhn, "The Hungarian method of solving the assignment problem," *Nav. Res. Logist. Quart*, vol. 2, no. 1-2, pp. 29–47, mar 2010.

[2] M. W. P. Savelsbergh and M. Sol, "The General Pickup and Delivery Problem," *Transp. Sci.*, vol. 29, no. 1, pp. 17–29, feb 1995.

[3] S. N. Parragh, K. F. Doerner, and R. F. Hartl, "A survey on pickup and delivery problems. Part I: Transportation between customers and depot," *J. für Betriebswirtschaft*, vol. 58, no. 2, pp. 81–117, 2008.

[4] J. F. Allen, "Maintaining knowledge about temproal intervals," *Readings Qual. Reason. about Phys. Syst.*, vol. 26, no. 11, pp. 361–372, nov 1990.

[5] M. Lagoudakis, M. Berhault, S. Koenig, P. Keskinocak, and A. Kleywegt, "Simple auctions with performance guarantees for multi-robot task allocation," *2004 IEEE/RSJ Int. Conf. Intell. Robot. Syst.*, vol. 1, pp. 698–705, 2004.

[6] D. Silver, "Cooperative Pathfinding." *AIIDE*, 2005.

[7] M. Cap, J. Gregoire, and E. Frazzoli, "Provably safe and deadlock-free execution of multi-robot plans under delaying disturbances," in *2016 IEEE/RSJ Int. Conf. Intell. Robot. Syst.*, vol. 2016-Novem. IEEE, oct 2016, pp. 5113–5118.

[8] H. Ma and S. Koenig, "Optimal Target Assignment and Path Finding for Teams of Agents," *Aamas 2016*, pp. 1144–1152, dec 2016.

[9] M. Koes, I. Nourbakhsh, K. Sycara, S. D. Ramchurn, and Others, "Heterogeneous multirobot coordination with spatial and temporal constraints," in *Proc. Natl. Conf. Artif. Intell.*, vol. 20, no. 3, 2005, p. 1292.